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PBA-003-001308

Seat No. ______

B. Sc. (Sem. III) (CBCS) Examination

November / December - 2018

Mathematics : Paper - BSMT - 301(A)

Faculty Code : 003

Subject Code : 001308

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instruction : All questions are compulsory.

1 Answer the following questions briefly : **20**

(1) Define linear combination of vectors.

(2) Define linear independence.

(3) If $T(x, y) = (x - y, 2x - y, 3x - 3y)$, then find $T(1,2)$.

(4) Define linear span.

(5) Define range of linear transformation.

(6) Find the Radius of curvature of $s = c \tan \psi$.

(7) Define Nilpotent linear transformation.

(8) If $p(x) = x^5 - 5x^4 - 7x + 8 = 0$, then find the value of p'(4).

(9) Write Cauchy's general principle of convergence of series.

(10) Check whether the series $\sum \frac{1}{n^2}$ is convergent or divergent ?

(11) Write Newton Raphson formula for finding root of $f(x) = 0$.

(12) Write Newton Raphson formula for $\frac{1}{N}$.

(13) Define curvature.

(14) Define singular point.

(15) Define point of inflexion.

(16) Write formula of radius of curvature in the pedal form.

(17) Define Node.

(18) Find asymptotes parallel to x–axis for the curve $4x^2 + 9y^2 = x^2 y^2$.

(19) Find the interval in which lies one real root of the equation $x^2 - x - 2 = 0$.

(20) Write D'Alembert Test.

2 (a) Answer any **three** : **6**

(1) Prove that $(1, 5, 4) \in$ Span $\{(1, 3, 0), (0, 1, -1), (0, 0, 2)\}$ is a subspace of R^3.

(2) Check whether set $\{ (1, 3, 2), (1, -7, -8), (2, -1, -1) \}$ is linearly dependent or not.

(3) Show that $\{(2, 2, 3), (2, 1, 3), (1, 0, 1)\}$ is a basis of R^3.

(4) Find Eigen values of linear transformation $T : R^2 \to R^2$, where $T(x, y) = (3x + y, 6x + 2y)$

(5) Find N_T, R_T of linear transformation $T : R^4 \to R^3$. where $T(a, b, c, d) = (a + b, b - d, c - d)$.

(6) Discuss the convergence of series $\sum_1^\infty n^4$.

(b) Answer any **three** : **9**

(1) Prove that any finite superset of a linearly dependent vector set is linearly dependent.

(2) Prove that the series $\sum_{n=1}^\infty \frac{n+2}{n^3+1}$ is convergent.

(3) If U and V are vector spaces, T : U $\to$ V is linear transformation and θ is zero vector of U, then $N_T = \{e\}$ if and only if T is one - one.

(4) Show that series $\sum \frac{n^2}{3^n}$ is convergent.

(5) Prove that the composition of two linear transformations is again a Linear transformation.

(6) Discuss the convergence of series $\sum_1^\infty \frac{n}{n^2+1}$.

(c) Answer any **two** : **10**

(1) Discuss the convergence of the series

$$\sum \frac{3.6.9..........3n}{7.10.13..........(3n+4)} x^n, x > 0.$$

(2) In usual notation prove that

$$\dim(W_1 + W_2) = \dim(W_1) + \dim(W_1) - \dim(W_1 \cap W_2)$$

(3) If $V = \{(x, y) / x > 0, y > 0, x, y \in R\}$ and for $(a, b), (c, d) \in V$ and $\alpha \in R, (a, b) + (c, d) = (ac, bd)$ and $\alpha(a, b) = (a^{\alpha}, b^{\alpha})$, then show that V is a vector space.

(4) State and prove Rank- Nullity theorem.

(5) Prove that { (2, 2, 3), (2, 1, 3), (1, 0, 1) } is a basis of R^3. Find coordinates of (1, 1, 1) with respect to this basis.

3 (a) Answer any **three** : **6**

(1) Find asymptotes of the curve $y = \frac{9}{x-5}$.

(2) Find asymptotes parallel to X - axis of curve $a^2(x^2 + y^2) = x^2y^2$.

(3) Find asymptotes parallel to Y - axis of curve $a^2(x^2 + y^2) = x^2y^2$.

(4) Find p at origin by Newton's method for the curve $x^3 + y^3 = 3axy$.

(5) Find points of inflexion of the curve $x^3 = y(3 + x^3)$.

(6) Find Radius of curvature of $y^2 = 12x$ at the point (0, 0).

(b) Answer any **three** : **9**

(1) Derive the formula for approximation value of $\sqrt{N}$.

(2) Explain Iterative method to obtain the approximate value of a real root of the equation $f(x) = 0$.

(3) Show that point (0, 0) is a node for the curve $x^4 + y^4 = 4axy$.

(4) Prove that for the cardioids

$r = a(1 - \cos\theta), \rho = \frac{4a}{3}\sin\frac{\theta}{2}$ at point θ.

(5) Find all the asymptote of the curve $x^2y^2 = x^2 - a^2y^2$.

(6) Find the singular points of the curve $xy^2 - (x+y)^2 = 0$.

(c) Answer any **two** : **10**

(1) Explain False position method for finding root of the equation $f(x) = 0$.

(2) Explain Horner's method to obtain the approximate value of a real root of the polynomial $f(x) = 0$ of degree n.

(3) Find x coordinate of point of inflexion of the curve $x^2y - 4x + 3y = 0$.

(4) Derive the formula of ρ for the polar coordinate system of the curve $r = f(\theta)$.

(5) Find radius of curvature of the curve $r^2 = a^2\cos\theta$ at θ point.